

Mathematical Formulas

Substitution of MDF into DUF to Obtain the IUF:

$$V(P, M) = U(x^M(P, M))$$

IUF vs. EF:

$$V(P, E(P, u)) = u \quad \text{AND} \quad E(P, V(P, M)) = M$$

Shephard's Lemma:

$$x_i^c = \frac{\partial E(P, u)}{\partial P_i}$$

DUF vs. DF:

$$U\left(\frac{q}{D(q, u)}\right) = u$$

Hotelling-Wold Identity:

$$\phi_i(q) = p_i = \frac{P_i}{M} = \frac{\frac{\partial U}{\partial q_i}}{\sum_{j=1}^n \frac{\partial U}{\partial q_j} \cdot q_j}$$

Antonelli Equation:

$$\psi_i(q, u) = p_i = \frac{P_i}{M} = \frac{\partial D(q, u)}{\partial q_i}$$

Slutsky Equation:

$$\frac{\partial x_i^M(P, M)}{\partial P_j} = \frac{\partial x_i^c(P, u)}{\partial P_j} - \frac{\partial x_i^M(P, M)}{\partial M} \cdot x_j(P, M)$$

Substitution of HDF into EAF to Obtain the EF:

$$E(P, u) = E(P, x^c(P, u))$$

OR

$$E(P, u) = P \cdot x^c(P, u)$$

Substitution of the Inverse of MDF into IUF to Obtain the DUF:

• Invert $x^M(P, M)$

to get $P(x^M)$

• $U(q) =$

$$V(P(x^M), M)$$

Substitution of the Inverse of HDF into EF to Obtain the EAF:

• Invert $x^c(P, u)$

to get $P(x^c)$

• $E(P, q) =$

$$E(P(x^c), u)$$

Roy's Identity:

$$x_i^M = - \frac{\frac{\partial V}{\partial P_i}}{\frac{\partial V}{\partial M}}$$

MDF vs. HDF:

$$x^c = x^M(P, E(P, u))$$

$$x^M = x^c(P, V(P, M))$$